BIDIMENSIONAL TURING MACHINES AS GALILEAN MODELS OF HUMAN COMPUTATION

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Overview (1/3)

- Even though *simulation models* are the dominant paradigm in cognitive science, it has been argued (Giunti 1997) that *Galilean models* might fare better on both the description and explanation of real cognitive phenomena.
- The main goal of my paper is to show that the actual construction of Galilean models is clearly feasible, and well suited, for a *special class* of cognitive phenomena, namely, those of human computation.

Overview (2/3)

- I argue in particular that Turing's original formulation of the Church-Turing thesis (1936, sec. 9.I.) can naturally be interpreted as the core hypothesis of a new empirical theory of human computation;
  - this theory relies on *bidimensional* Turing machines, a generalization of ordinary machines with one-dimensional tape to machines working on two-dimensional paper;
  - the thesis I argue for is that such machines can be used to construct *Galilean models* of the phenomena of human computation.
- Finally, I also suggest that this new theory of human computation might become a first paradigm for a *general* approach to the study of cognition, an approach entirely based on Galilean models of cognitive phenomena.

Overview (3/3)

- However, because of time restrictions, this talk will be limited to:

1. outline the distinction between simulation models and Galilean models;
2. give some hints as to why, in general, the second ones fare better on both the *descriptive* and the *explanatory* score;
3. make clear what "phenomenon of human computation" means;
4. finally, suggest how (bidimensional) Turing machines can be thought as potential Galilean models of the phenomena of human computation.

Simulation models

- A *simulation model of a real phenomenon H* is a mathematical dynamical system that
  - is implemented on a digital computer by means of appropriate software and
  - allows us to produce empirically correct simulations of *H*.
- A simulation is empirically correct if we are able to empirically establish that the simulating process is similar to *H* in some relevant respect;
  - which respects are to be considered relevant, and which empirical methods we may employ to establish the similarity, is usually clear in each specific case.

Simulation models – Descriptive limit

- The descriptive limit concerns the correspondence between simulation data and real ones, which is not direct and intrinsic to the model, but at most indirect and extrinsic.
- For, typically, a simulation model does not incorporate measurable properties (magnitudes) of the real phenomenon among its basic components;
  - in contrast, quantitative descriptions are typically obtained by just matching such properties with superficial, global, or emergent features of the model.

Galilean models

- As a first approximation, we can think of a Galilean model as a dynamical system with *n* ($1 \leq n$) state components, where each component has a precise and definite empirical interpretation, as it corresponds to a measurable magnitude of the real phenomenon that the model describes.
- A more precise characterization of a Galilean model presupposes a preliminary analysis of the very notion of a phenomenon, but it exceeds the scope of this presentation.
- Example. Galilean model of free fall.

Simulation models – Explanatory limit

- The explanatory limit concerns the quality of the explanations supported by the model. Typically, they are neither realistic nor comprehensive, as they are rather cast in a somewhat fictional and "in principle" style.
- This second limit, like the first one, is due to the fact that the basic components of a simulation model do not directly correspond to real aspects of the phenomenon itself.
- As a consequence, any explanation that is based on analyses of a model of this kind is bound to introduce a whole series of fictional characters, which do not have any real counterpart in the phenomenon.

Galilean models – Better descriptions and explanations

- Galilean models can go well beyond the descriptive and explanatory limits of simulation models.
- Note first that data description in Galilean models is direct and intrinsic, for each component of a model of this kind determines the values of a specific magnitude of the corresponding phenomenon.
- Second, the explanations supported by a Galilean model are realistic and comprehensive, as each of its components corresponds to a specific magnitude of the intended phenomenon, and so any explanation based on an analysis of such a model cannot introduce any arbitrary or fictional character.

Phenomena of human computation

- In general, by a *phenomenon of human computation* we mean any activity of a human being that consists in executing a purely mechanical or effective procedure,
- where a *mechanical procedure* is a finite set of clear-cut formal instructions for symbol manipulation;
  - given a finite series of data, a human being must be able to carry out such instructions in a definite sequence of steps, with the exclusive aid of paper and pencil (or equivalent external devices), and without resorting to any special insight or ingenuity.

The three state variables of any phenomenon of human computation

- Alan Turing (1936, sec. 9.I, 250) explicitly pointed out that *any* phenomenon of human computation is completely described by the time evolution of exactly *three* different magnitudes (the *state variables* of the phenomenon), namely:

1. the whole content of the paper on which the human being carries out the calculations;
2. the exact location of the symbols observed by the human being;
3. the human being's state of mind (relative to the calculation).

Ordinary Turing machines – General description

- An ordinary Turing machine can be thought as a device formed by a *head*, just one slot of *internal memory*, and a linear *tape* (*external memory*) divided into adjacent squares.
- The internal memory slot always contains exactly one symbol (*internal state*) taken from a finite alphabet $Q = (q_1, \dots, q_m)$ with at least one element;
- similarly, each tape square contains exactly one symbol taken from a second finite alphabet $A = (a_0, a_1, \dots, a_n)$ with at least two elements.
- The head is always located on exactly one square of the tape (*the scanned square*), and it is capable of performing five basic operations: *read* the symbol on the scanned square, *write* a new symbol on such a square, move one square to the *right* (indicated by *R*), move one square to the *left* (*L*), *stay put* (*H*).

Ordinary Turing machines – The three state variables

- For our present purposes, it just suffice to say that the complete behavior of an ordinary Turing machine is fully described by the time evolution of the following three variables (*the machine's state variables*):

1. the whole content of the machine tape;
2. the location of the scanned symbol;
3. the internal state of the machine.

Bidimensional Turing machines – The three state variables

- Bidimensional Turing machines are obtained by just replacing the linear tape of ordinary machines with a two dimensional checkerboard, potentially infinite both in the right/left direction and in the upward/downward one.
- Accordingly, the head can now also move one square up (*U*) and one square down (*D*).
- As before, the complete dynamic behavior of the machine is still described by exactly three state variables, which now are:

1. the whole content of the machine checkerboard;
2. the location of the scanned symbol;
3. the internal state of the machine.

Upshot – Any (bidimensional) TM is a potential Galilean model of any given phenomenon of human computation

- Given any phenomenon of human computation *H*, and any (bidimensional) Turing machine *TM*, *TM* is a *potential Galilean model of H*.
- For there is a *natural* one-to-one mapping between the state-variables of the machine *TM* and the state-variables of the real phenomenon *H*. Namely,
  - checkerboard content → paper content
  - scanned symbol location → observed symbol location
  - internal state → mental state

THAT'S ALL

THANK YOU

References

Giunti, Marco (1997), *Computation, Dynamics, and Cognition*. New York: Oxford University Press.

Turing, Alan M. (1936), "On computable numbers, with an application to the Entscheidungsproblem", *Proceedings of the London Mathematical Society* ser. 2, 42:230-265.

Galilean model of free fall

- As an example, let us consider the following system of two ordinary differential equations $\langle dx(v)/dv = \boldsymbol{k},\ dy(v)/dv = x(v)\rangle$, where $\boldsymbol{k}$ is a fixed real positive constant. The solutions of such equations uniquely determine the dynamical system $DS_e = (X \times Y, (h^v)_{v \in V})$, where $X = Y = V = R$ (the real numbers) and, for any $v, x, y \in R$, $h^v(x, y) = (\boldsymbol{k}v + x,\ \boldsymbol{k}v^2/2 + xv + y)$.
- On the other hand, let us consider the phenomenon of free fall H_e, and let IH_e be the following interpretation of DS_e on H_e. The first component X of the state space of DS_e is the set of all possible values of the *vertical velocity* of an arbitrary free falling body, the second component Y is the set of all possible values of the *vertical position* of the falling body, and the time set V of DS_e is the set all possible instants of *physical time*.
- Since all three of these magnitudes are measurable or detectable properties of the phenomenon of free fall H_e, IH_e is an empirical interpretation of DS_e on H_e, and (DS_e, IH_e) is thus an empirical model of H_e. For an appropriate value of the constant $\boldsymbol{k}$, such a model also turns out to be empirically correct. Then, the pair (DS_e, IH_e) is said *a Galilean model of H_e*.